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ABSTRACT

A project concerned with the modification of a mathematical probability and statistics course is described. The course was redesigned to include a weekly two hour laboratory session in which students use an IBM 1130 computer to verify empirically the theoretical proofs learned in the classroom. This project report lists the subroutines furnished to the students and the topics of each of the 15 lab sessions. In addition, descriptions of three laboratory exercises are given by way of illustration, followed by a summary of student reactions and faculty attitudes toward the use of the computer for laboratory work. (LB)

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A COMPUTER LABORATORY FOR MATHEMATICAL PROBABILITY AND STATISTICS*

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For a number of years the junior-senior level mathematical probability and statistics course at Hope College was a two semester course meeting for lecture and discussion three periods per week. This project* concerns itself with a modification of the course so that a laboratory which meets two hours each week is an integral part of it. One hour of credit is given each semester for this laboratory which makes extensive use of our IBM 1130 computer.

The development of the laboratory materials is taking place over a 27 month period, June, 1971, through August, 1973. At the 1972 Conference on Computers in the Undergraduate Curricula this project was described and a listing of the laboratories for the first semester was given [1]. We are now able to list the laboratories for the second semester, describe some of them, and give a better overall evaluation of the laboratory.

Laboratory Description

In the laboratory the students illustrate empirically many of the theoretical concepts which are proved in the classroom. By writing computer programs they gain a better and deeper understanding of the theory. In addition they become aware of the capabilities of the computer for analyzing data.

We generally spend the first hour of the laboratory discussing the assignment of the previous week and introducing the new assignment. The second hour of the laboratory is used to design and write the program. The format of the laboratory would probably be different if the students had access to terminals.

Several subroutines are furnished for the students. These include:

(1) a random number generator,

(2) plotting subroutines for (a) empirical and theoretical distribution functions, (b) relative frequency histograms and probability density functions, and (c) scatter diagrams with the option of adding the least squares regression line,

(3) programs which give values of a number of distribution functions, and

(4) subroutines which select random samples from the standard distributions.

The following listing gives the topics covered in each of the 15 laboratories during the second semester. In section 3 we shall describe three experiments.

Laboratory	Topics
1	Transformations of Random Variables, Cauchy Distribution, N(0,1) Distribution Using Box-Muller Method.
2	Chi-square and Beta Distributions.
3	Functions of Independent and Identically Distributed Normal Random Variables.
4	The t and F Distributions.
5	Convergence in Probability; Convergence in Distribution.
6	Central Limit Theorem.
7	Normal Approximations for the Binomial Distribution.
8	Estimation.
9	Confidence Intervals for Means and Variances.
10	Confidence Intervals for Proportions.
11	Tests of Simple Hypotheses.
12	The Power Function.
13	Analysis of Variance.
14	A Test of the Equality of Multinomial Distributions.
15	Order Statistics.

Laboratory Examples

We shall give a brief description of parts of three laboratories.

Laboratory 4 - The t and F Distributions: A t random variable with r = 8 degrees of freedom is defined by

$$T = \frac{Z}{\sqrt{U/8}}$$

where Z has a normal distribution with mean $\mu=0$ and variance $\sigma^2=1$ [hereafter N(0,1)], U has a chi-square distribution with r=8 degrees of freedom [hereafter $\chi^2(8)$], Z and U are independent.

The students are able to simulate samples from both the normal and chi-square distributions. Thus they are able to simulate n=300 observations of T and obtain a histogram similar to that shown in Figure 1. This is a relative frequency histogram with the probability density function for the t distribution with r=8 degrees of freedom superimposed.

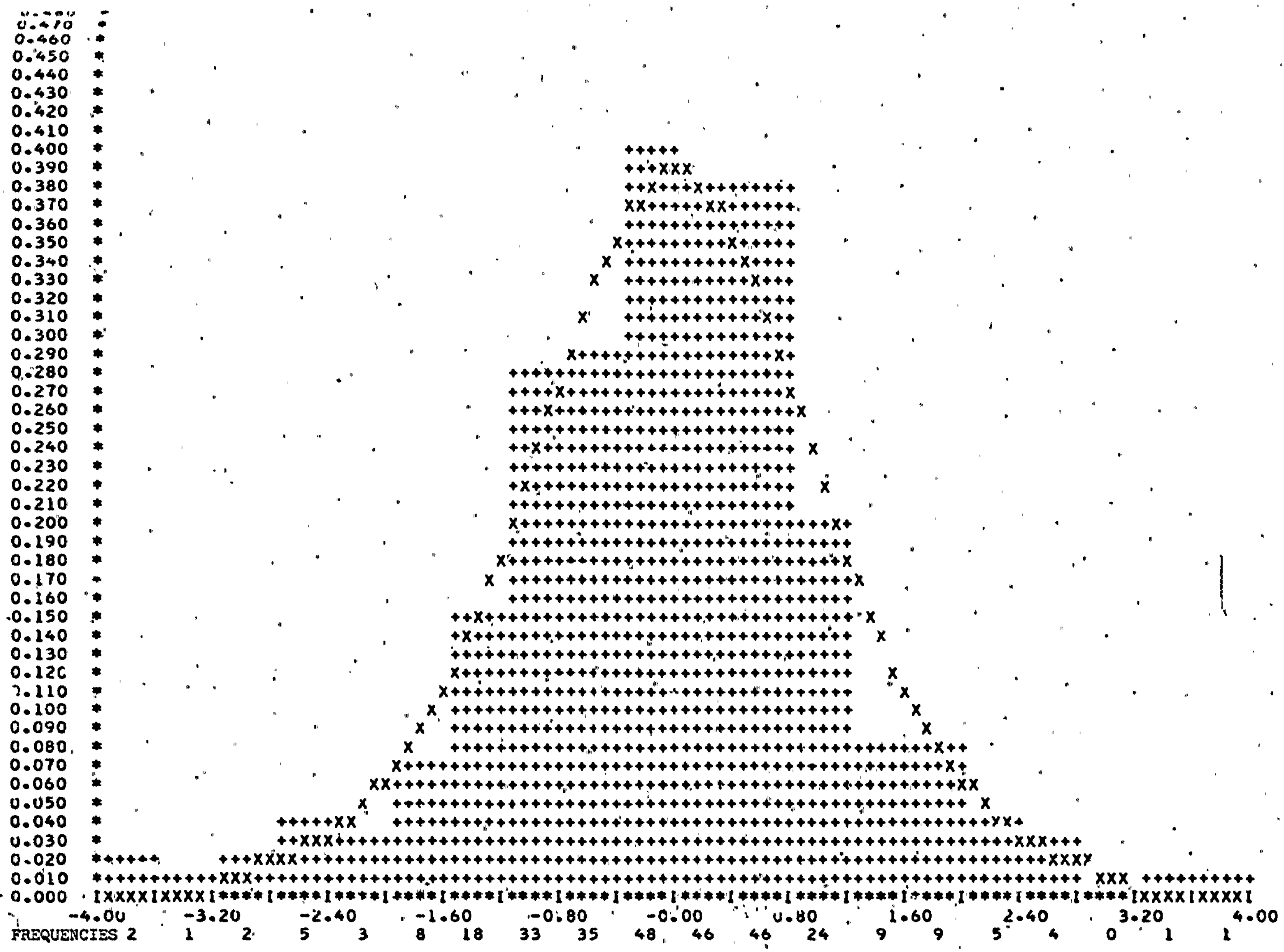
The importance of the t distribution is its use in normal sampling. To illustrate this, let $X_1, X_2, \ldots, X_9$ be a random sample from a distribution which is N(30,16). Then

$$T = \frac{\bar{X}-30}{\sqrt{\dfrac{\sum (X_i-\bar{X})^2}{(9)(8)}}}$$

has a t distribution with r=8 degrees of freedom. A random sample of size n=300 observations of T, generated using this relationship, yielded the histogram in Figure 2.

Students are able to raise and partially answer questions concerning the hypotheses of certain theorems. For example, what if the random sample in the above exercise does not come from a normal distribution. Does T defined above still have a t distribution?

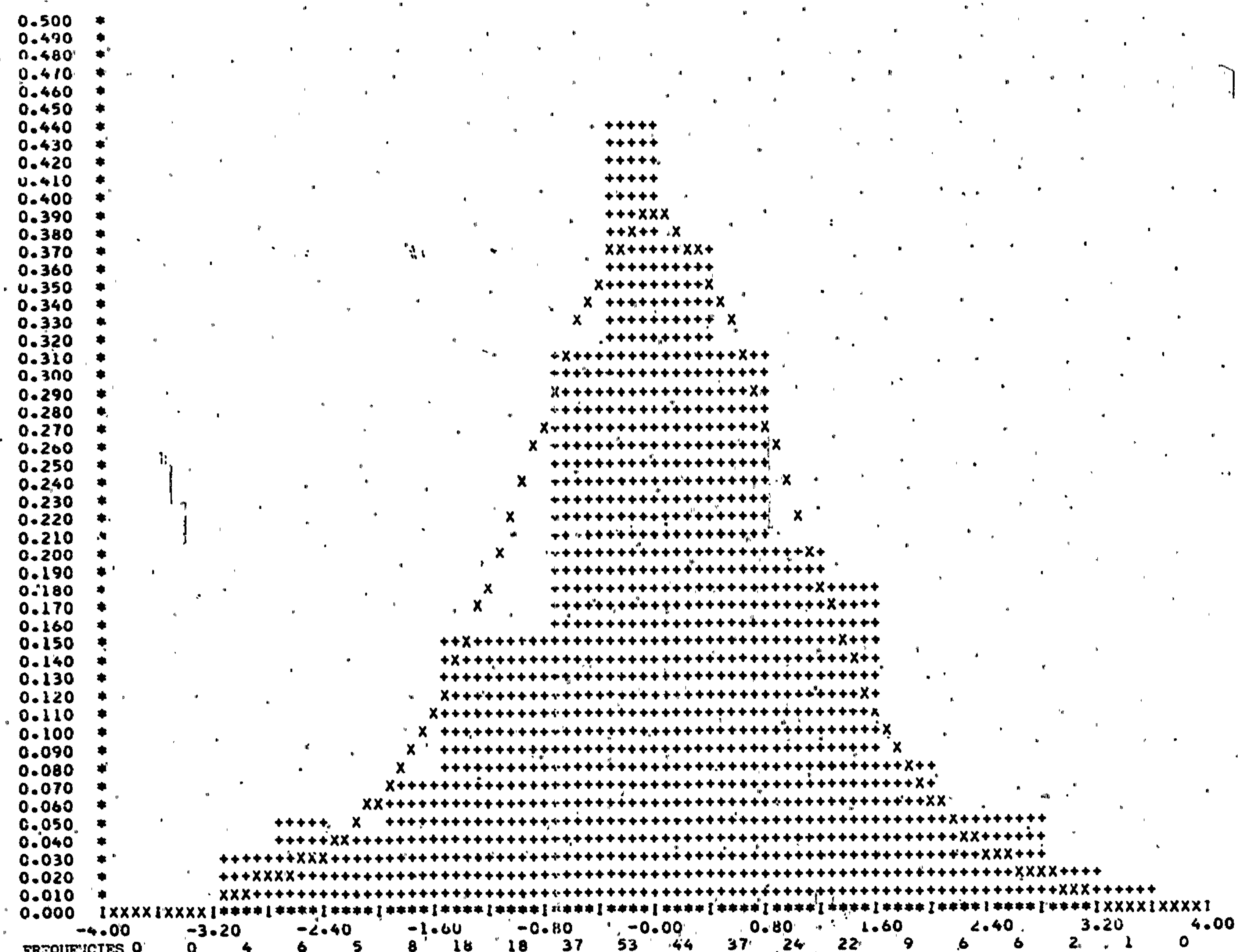
Or suppose the sample does come from a normal distribution. What effect does rounding off the observations have on the distribution of T? This question was investigated by one of



THE SAMPLE MEAN AND SAMPLE VARIANCE ARE -0.0716 AND 1.2668

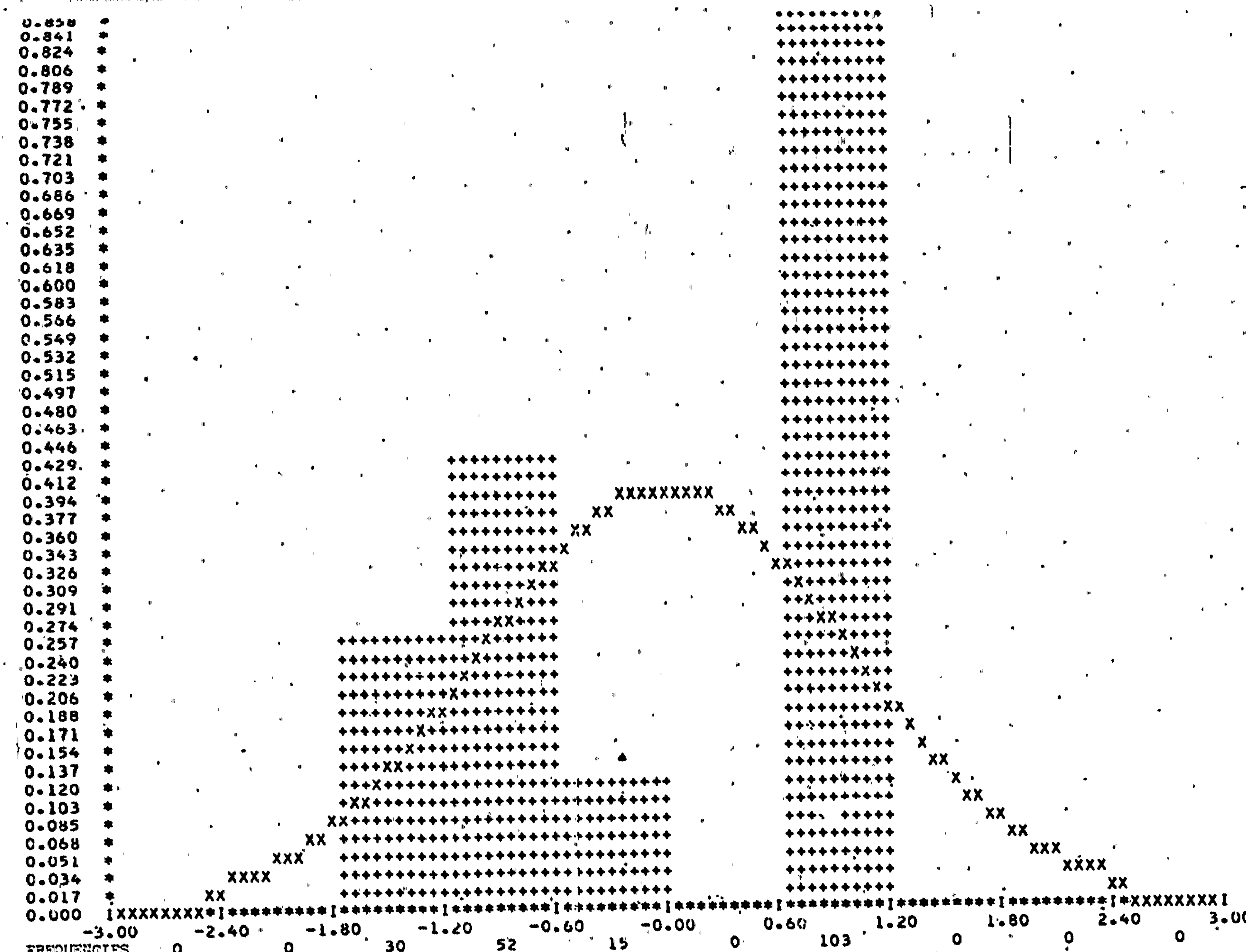
PDF FOR A T DISTRIBUTION WITH 8 DEGREES OF FREEDOM.

FIGURE 1



THE SAMPLE MEAN AND SAMPLE VARIANCE ARE 0.0362 AND 1.3357

PDF FOR A T DISTRIBUTION WITH 8 DEGREES OF FREEDOM.



THE SAMPLE MEAN AND SAMPLE VARIANCE ARE 0.0174 AND 1.0092

N = 1

FIGURE 3

our students using different combinations of the variance, sample size, and round off accuracy [2].

Laboratory 6 - The Central Limit Theorem: (This particular example was motivated by the work of two Hope College students [3,4].)

This experiment illustrates the Central Limit Theorem and gives students some feeling for deciding when n is "sufficiently large" for using the normal approximation. This particular example also helps to emphasize that the Central Limit Theorem is applicable to continuous, discrete, and mixed distributions.

Consider the following game. Flip a coin. If the outcome is heads, win $2; if the outcome is tails, spin a balanced spinner which has outcomes between 0 and 1 with the payoff equal to the outcome on the spinner. Let X denote the payoff for this game. Then X has a mixed distribution, $\mu=5/4$, $\sigma^2=29/48$. Let $X_1, X_2, \ldots, X_n$ denote the outcomes on n independent plays of this game. Then

$$W_n = \frac{\bar{X} - 5/4}{\sqrt{29/(48n)}}$$

converges in distribution to the random variable W which is N(0,1).

In the laboratory we would illustrate that W_n has a distribution which is approximately N(0,1) when n is sufficiently large. In this paper we shall use two values of n, n=1 and n=13. This game was simulated 200 times yielding 200 observations of W_1. The histogram of these data is plotted in Figure 3 with the probability density function for the normal distribution, $\mu=0$, $\sigma^2=1$ superimposed. An observation of the random variable W_{13} is simulated using 13 independent plays of the game. Thus 200 observations of W_{13} are based upon 2600 plays of the game. The histogram of 200 observations of W_{13} is given in Figure 4 with the N(0,1) probability density function superimposed. The empirical distribution function for the first 25 observations of W_{13} is plotted along with the N(0,1) distribution function in Figure 5.

Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from a distribution which has mean μ and variance σ^2. Let

$$W_n = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}.$$

A "rule of thumb" given in many textbooks is that W_n has a distribution which is approximately N(0,1) when n is greater than 25 or 30. Using the techniques learned in the above and similar exercises, a student is able to test this "rule of thumb" empirically for serveral distributions.

Laboratory 12 - The Power Function: This laboratory illustrates the power function both theoretically and empirically. We shall describe the particular exercise which uses a test on the variance.

Let X have a normal distribution with mean μ and variance σ^2. We shall test the null hypothesis $H_0: \sigma^2=47.5$ against the alternative hypothesis $H_1: \sigma^2<47.5$.

If the test is based upon a sample of size n=10, a critical region for a significance level of

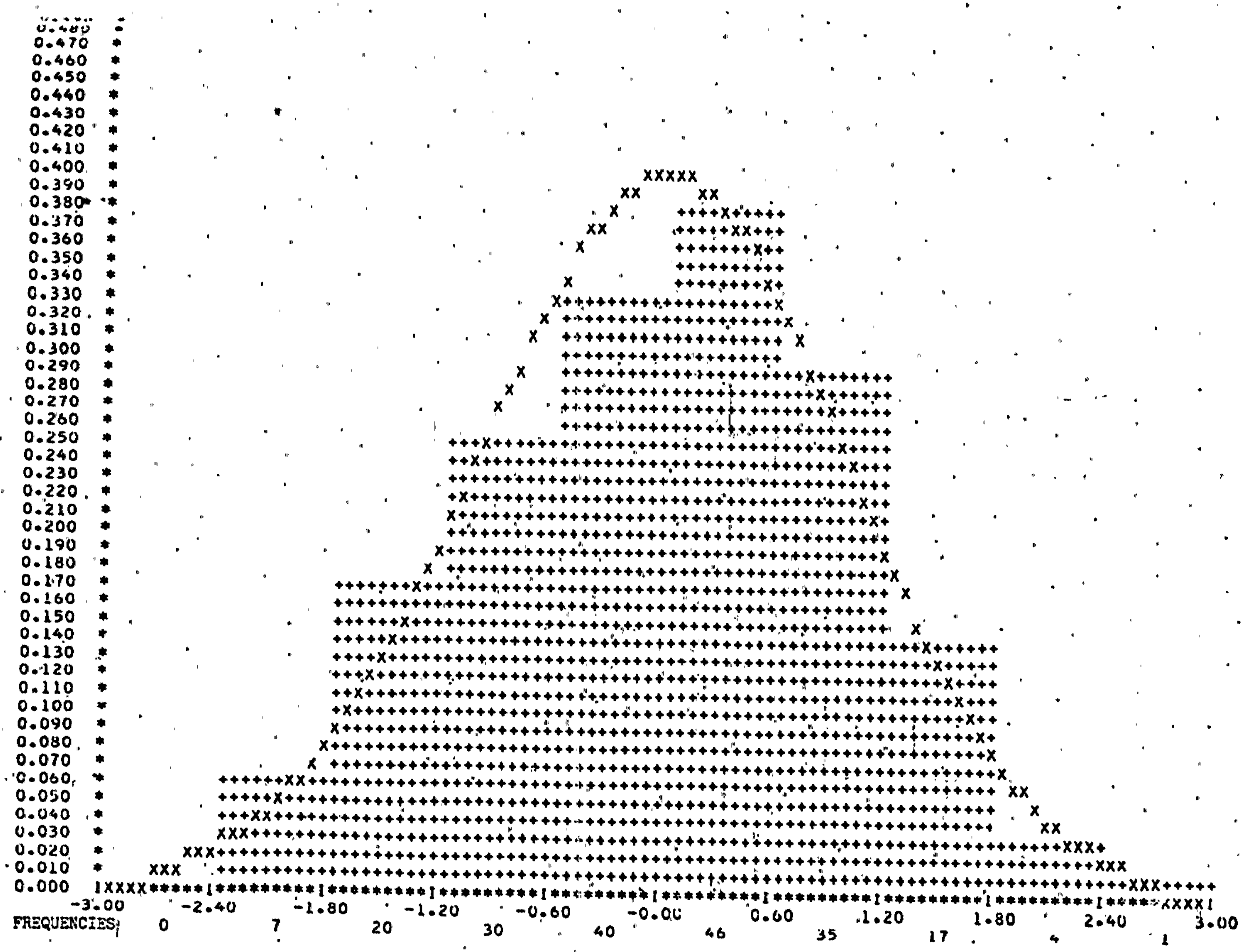
$\alpha=0.05$ is $C=\{(x_1, x_2, \ldots, x_{10}): \sum_{1}^{10}(x_i - \bar{x})^2/47.5 < 3.325\}$.

The power function for this test is defined by

$$K(\sigma^2) = P\left(\frac{\sum_{1}^{10}(X_i-\bar{X})^2}{\sigma^2} < \frac{(3.325)(47.5)}{\sigma^2}\right)$$

where $\sum_{1}^{10}(X_i-\bar{X})^2/\sigma^2$ has a distribution which is $\chi^2(9)$. A subroutine for the chi-square distribution function permits us to graph this power function.

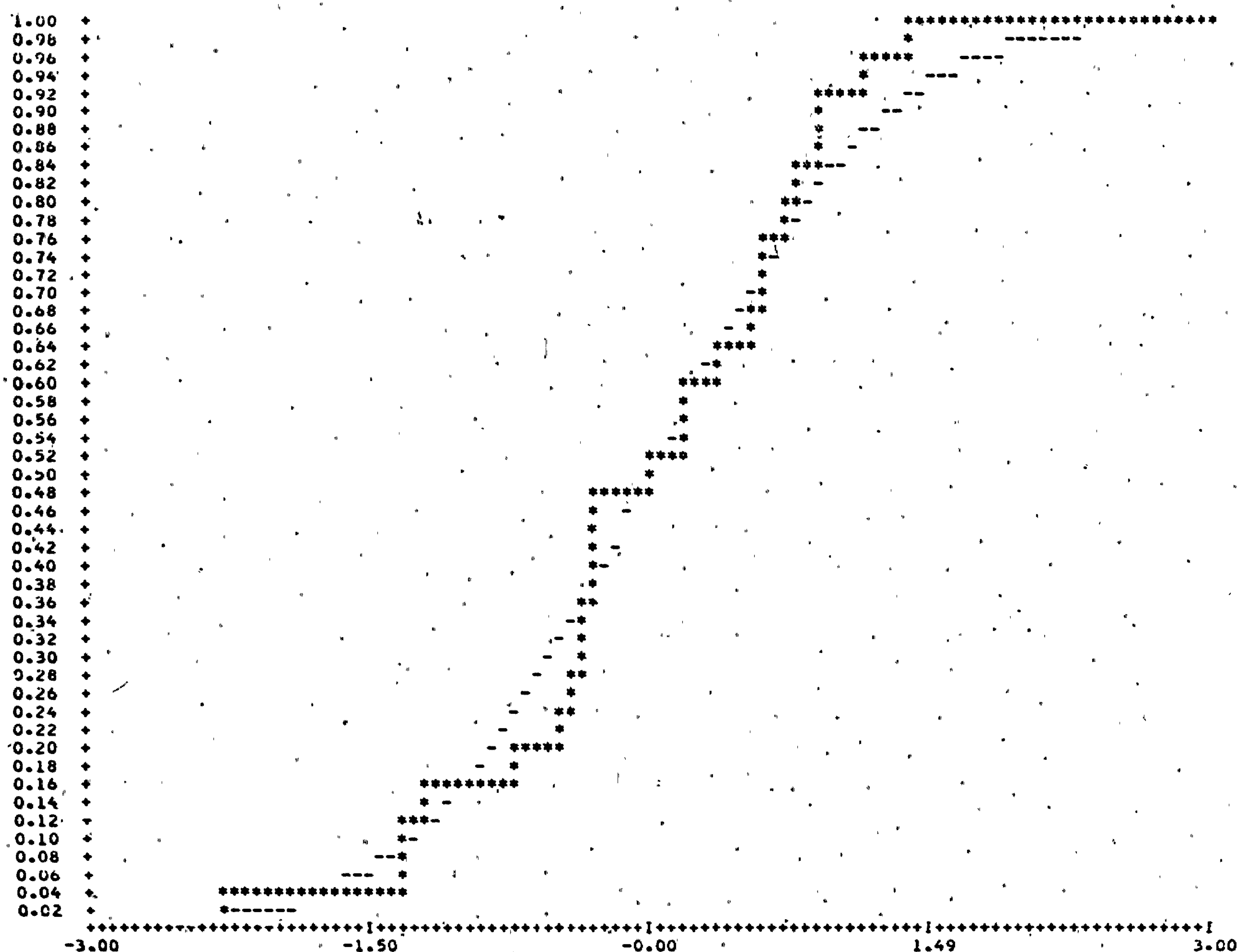
To illustrate the power function empirically, for $\sigma^2=2.5, 47.5, 5$, we generated 50 random samples of size 10 from a distribution which was $N(0,\sigma^2)$. For each of the 10 values of σ^2, we counted the number of times that H_0 was rejected. The relative frequency of rejections for a particular value of σ^2 should be close to $K(\sigma^2)$.



THE SAMPLE MEAN AND SAMPLE VARIANCE ARE -0.0049 AND 0.9422

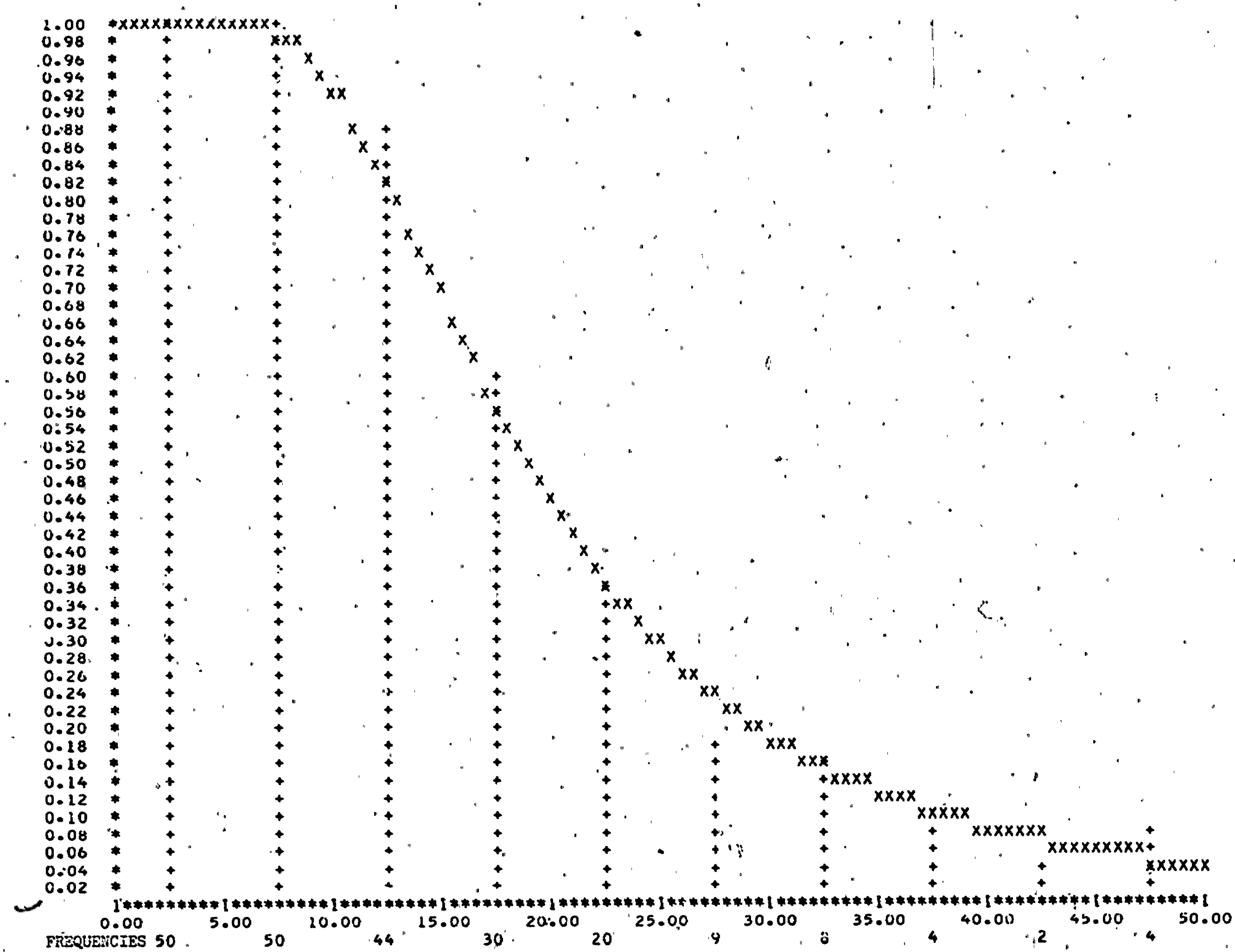
N = 13

FIGURE 4



THE MAXIMUM DIFFERENCE BETWEEN THE THEORETICAL
AND THE EMPIRICAL CURVES IS 0.109

FIGURE 3


1.00
0.98
0.96
0.94
0.92
0.90
0.88
0.86
0.84
0.82
0.80
0.78
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0.74
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0.10
0.08
0.06
0.04
0.02
0.00 5.00 10.00 15.00 20.00 25.00 30.00 35.00 40.00 45.00 50.00
FREQUENCIES 50 50 44 30 20 9 8 4 2 4

FIGURE 6

In Figure 6 the power function is graphed along with the relative frequencies of rejections for the 10 values of σ^2.

With the techniques learned in this example, a student is able to compare two power functions for the same test which differ only in the size of the critical region. A student is also able to define a power function empirically when he is perhaps not able to define it theoretically.

Student Reaction

This laboratory has been taken by two classes of junior and senior mathematics and science majors. The general reaction of most of the students to the laboratory has been favorable. Of the 25 students in this year's class who responded to the question "Did the laboratory help you understand more clearly the theoretical concepts?", 18 said yes, 4 said no, and 3 said somewhat.

One student thought the laboratory was very helpful in highlighting the material in class. Another said it took a lot of time but was interesting and helpful. Still another student felt that many of the theoretical principles of statistics were reinforced by the lab work. A few students suggested that the use of more data which had been collected in science laboratories would have been interesting. For one student the laboratory was not beneficial because she generally believed the theoretical concepts without the empirical printout.

I believe that in general the reaction of a student to the laboratory depends largely upon his like or dislike of the computer.

Faculty Evaluation

August 14-18, 1972, we held a conference at Hope College for 51 college teachers of statistics from the midwest and east. During this conference we described our use of the computer in our statistics curriculum. From this conference we received both written and verbal comments, some of which I would like to summarize.

It is not always possible to add a laboratory because of insufficient staff time or inadequate computer facilities. In such cases it is perhaps possible for the students to write only a few computer programs while the instructor illustrates most of the ideas with printouts or transparencies. However mathematics majors might be less intrigued with just seeing others' results without actually being able to perform the exercises on their own. Also students seem to gain intuition when they use the computer.

A valuable by-product of the statistics laboratory is the opportunity for students to do some research as undergraduates. Their mathematical maturity and background may not be sufficient to prove certain results theoretically but they are able to obtain answers empirically.

Some of our conference participants are using part of our materials during the 1972-73 academic year. We hope to receive summaries of their experiences by the end of this academic year.

Summary

A computer laboratory in statistics provides new and interesting challenges to both student and teacher. A lot of extra time is required. However if the exercises are meaningful and not just busywork, the time is well spent.

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